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Euclidian response of light nuclei

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Abstract. We discuss the longitudinal and transverse quasi-elastic cross-sections for ^{3}He and ^{4}He in terms of the Euclidian response, which can be directly calculated from the ground-state wave function. We find that the main open problem in quasi-elastic scattering, the excess of transverse strength, can be understood as a consequence of meson exchange currents, once the n-p short-range and tensor correlations in both initial and final state are included.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 25.30.-c Lepton-induced reactions

1 Introduction

Inclusive quasi-elastic electron-nucleus scattering has a number of interesting facets relating to properties of the nuclear spectral function (Fermi momenta, high-momentum components), the role of final-state interactions, y-, ψ- and ξ-scaling, superscaling, enhancement of the response in the "dip" region between quasi-elastic and Δ peak. Here, we want to focus on a different aspect: the integrated strength of the longitudinal and transverse responses and the role played by meson exchange currents.

The response functions R_L and R_T at constant 3-momentum transfer q are defined by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'} = \sigma_{\mathrm{Mott}} \left(\frac{Q^4}{q^4} R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + tg^2\frac{\theta}{2} \right) R_T(q,\omega) \right).$$

With the definition of

$$\Sigma(q,\omega,\epsilon) = \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'} / \sigma_{\mathrm{Mott}} \epsilon \left(\frac{q}{Q} \right)^4$$

the R's are obtained from a straight-line fit

$$\Sigma(q,\omega,\epsilon) = \epsilon R_L(q,\omega) + \frac{1}{2} \left(\frac{q}{Q} \right)^2 R_T(q,\omega)$$

as a function of the virtual-photon polarization

$$\epsilon = \left(1 + \frac{2q^2}{Q^2} tg^2 \frac{\theta}{2} \right)^{-1}.$$

In the past, much attention has been devoted to the Coulomb sum rule

$$R_L(q,\omega) = \sum_{f \neq 0} \left\langle f|\rho(q)|0 \right\rangle^2 \quad \delta(\omega + E_0 - E_f)$$

which, under simplifying assumptions, should yield

$$C_L(q) = \int_{\omega+}^{\infty} R_L(q,\omega)\mathrm{d}\omega = Z\tilde{G}_{ep} + N\tilde{G}_{en}.$$

These assumptions comprise: 1) q has to be big enough to eliminate Pauli blocking (q should be significantly bigger than $2k_{\mathrm{F}}$); 2) the contribution of elastic scattering is small; 3) meson exchange currents (MEC) are negligible; 4) relativistic effects are covered by the use of $\tilde{G}$; 5) the scattered electron can be described by plane waves, and 6) the integral extends to $\omega = \infty$. While assumptions 2) and 4) are safe, the other assumptions are not fulfilled and need calculated corrections.

In the past, the analysis of quasi-elastic data taken at Bates and Saclay seemed to yield a Coulomb sum that amounted to 60% of the expected one only [1]. The careful work of J. Jourdan [2] showed that this lack of strength was due to a number of omissions and errors: use of the wrong nucleon form factor (dipole instead of the experimentally known one), neglect of Coulomb corrections, omission of the relativistic corrections, lack of correction for finite ω_{max}, all of which go in the same direction. When doing things properly, the Coulomb sum rule for the data of [1] is fulfilled within the experimental uncertainty of 20%. When adding to the data the cross-sections taken at SLAC —which are much more sensitive to R_L as they are taken at very small angle— one finds that the Coulomb sum is fulfilled for $A = 12$, 40, 56 (where enough data are available) within the experimental uncertainty of 10%.

This apparent problem with the Coulomb sum has deflected attention from the *real* problem, the fact that the transverse strength is too large. While it has been long known that there is an excess in the transverse strength in the "dip", the work on superscaling [3] emphasized in the most clear way that already in the main quasi-elastic

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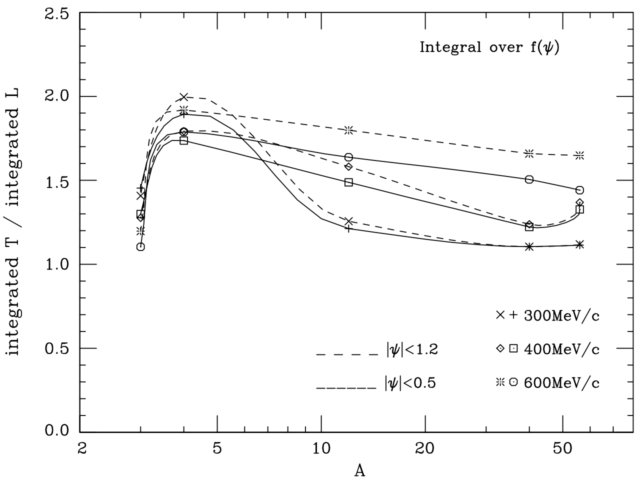

Fig. 1. Ratios of T and L strength integrated over two different regions of the quasi-elastic peak (centered at $\psi = 0$).

peak —below pion production threshold— R_T is much too large. This fact is best demonstrated by plotting the ratio C_T/C_L which, in the limit of the impulse approximation, should be roughly $= 1$ (neglecting the small convection current contribution to R_T). As shown by fig. 1 a significant excess of the transverse strength is observed. This excess is particularly pronounced for $A = 3$, 4.

This excess is not understood. A large number of calculations of MEC have been performed [4–17], but they in general find very small ($\leq 10\%$) contributions, with the exception of [13], who use medium-modified Δ properties adjusted to other MEC-sensitive quantities.

As mentioned above, the excess is particularly large for $A = 4$, and doubles between $A = 3$ and 4. For these nuclei very accurate experimental R's can be determined, and much more quantitative theoretical calculations can be performed. We, therefore, in the following want to pursue this question of the role of MEC for the helium nuclei.

2 Experimental response

In order to determine the longitudinal (L) and transverse (T) responses, the (e, e') world data on ^{3}He and ^{4}He have been analyzed [18]. A determination of the response functions from the world cross-section data [19–27] has many advantages over the traditional approach of using data from a single experiment only; the range in ϵ is much larger, thus allowing a more accurate separation of L and T. Particularly, for medium-A nuclei the limitations of the traditional approach were partly responsible for the misleading conclusions mentioned in the introduction and discussed in [2].

This separation has been done for values of q between 300 and 700 MeV/c. In fig. 2 we show as an example the L and T response (already divided by the proton charge form factor) for $q = 600$ MeV/c. While the longitudinal response approaches zero at large energy loss, the transverse response displays a rise due to the Δ which increases with increasing q.

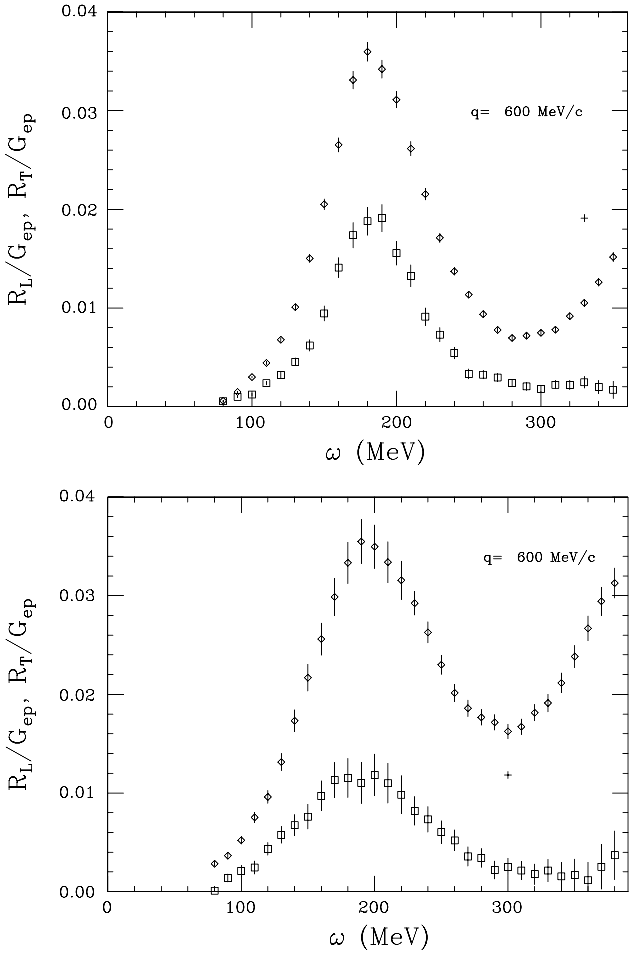

Fig. 2. T (◇) and L (□) response of ^{3}He (top) and ^{4}He at $q = 600$ MeV/c, already divided by the proton charge form factor. Crosses indicate the $\omega_{\max}$ used.

3 Euclidian response

A quantitative treatment of (e, e') requires a precise description of *both* the initial bound and the final continuum state. The latter is not available for $A > 3$. As an alternative, we study an integral over the response

$$\tilde{E}_{T,L}(q,\tau) \; = \; \int_{\omega_{\rm th}}^{\infty} \exp[-\omega\tau] \;\; R_{T,L}(q,\omega) \, {\rm d}\omega \, .$$

Here, E_0 is the ground-state energy of the nucleus, and $\omega_{\rm th}$ is the threshold for the response of the system excluding the elastic contribution. The longitudinal and transverse Euclidean response functions represent weighted sums of the corresponding $R_L(q,\omega)$ and $R_T(q,\omega)$: at $\tau = 0$ they correspond to the Coulomb and transverse sum rules, respectively, while their derivatives with respect to τ evaluated at $\tau = 0$ correspond to the energy-weighted sum rules. Larger values of τ correspond to integrals over progressively lower-energy regions of the response.

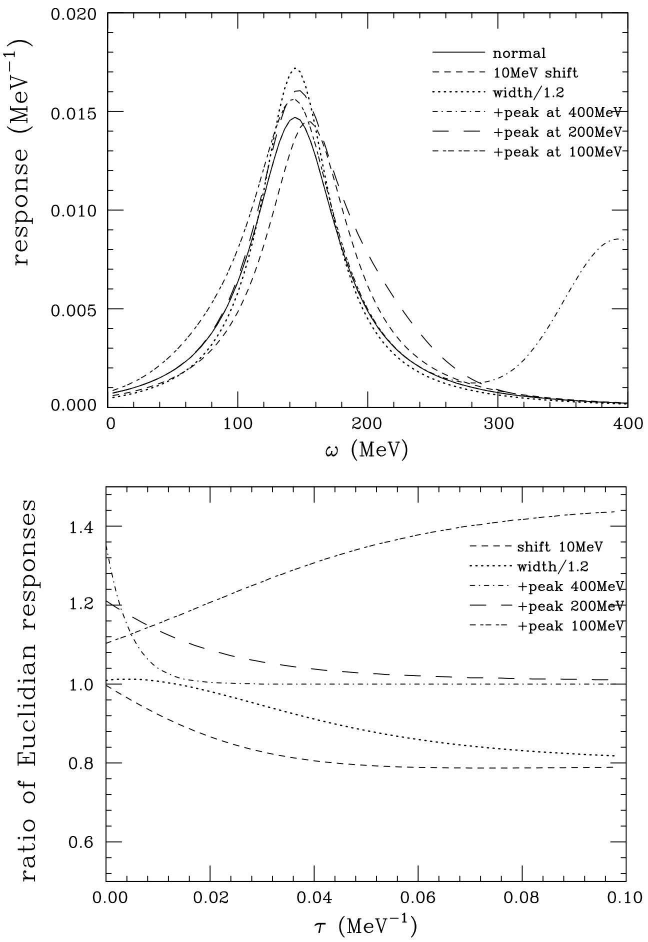

Fig. 3. Various modifications of the "normal" response (top) and the corresponding ratio to the unmodified Euclidian response (bottom).

In a non-relativistic picture, the $E_{T,L}$ can be simply obtained from:

$$\tilde{E}_L(q,\tau) = \langle 0|\rho^\dagger(\mathbf{q})\exp[-(H-E_0)\tau]\rho(\mathbf{q})|0\rangle - \exp\left(-\frac{q^2\tau}{2Am}\right)|\langle 0(\mathbf{q})|\rho(\mathbf{q})|0\rangle|^2,$$

and similarly for $\tilde{E}_T(q,\tau)$, with the charge operator $\rho(\mathbf{q})$ replaced by the current operator $\mathbf{j}_T(\mathbf{q})$. The elastic contributions have been explicitly subtracted, and $|0(\mathbf{q})\rangle$ represents the ground state recoiling with momentum $\mathbf{q}$.

The study of the Euclidian response has the outstanding advantage that $E(q,\tau)$ can be calculated from the ground-state properties alone; no explicit treatment of the final continuum state is required. For the $A = 3$, 4 ground states, very precise wave functions are available, and the effects of MEC can be included using the two-body operators well established in elastic and inelastic electron scattering from light nuclei (for a review see [28]).

The Euclidian response has the disadvantage that we usually lack a good feeling for this integrated quantity. Model studies [18] show the sensitivity of $E(q,\tau)$ to properties of $R(q,\omega)$, see fig. 3. The top panel shows various modifications of the "normal" response, the bottom panel shows the effect upon the ratios of the resulting Euclidian responses to the "normal" one. These studies show that, for the responses that can be extracted from the data, the region $0.01 \leq \tau \leq 0.05$ is the most relevant one for a comparison with theory.

The ground-state wave functions used in this study are obtained with variational Monte Carlo [18]. The Hamiltonian used is the Argonne v_8 N-N interaction plus the UIX three-nucleon interaction.

The one-body electromagnetic operators have the standard expressions obtained from a relativistic reduction of the covariant single-nucleon current. The two-body current operator consists of "model-independent" and "model-dependent" components, in the classification scheme of Riska [29]. The model-independent terms are obtained [30] from the nucleon-nucleon interaction. For the model-dependent pieces, the calculation includes the isoscalar $\rho\pi\gamma$ and isovector $\omega\pi\gamma$ transition currents as well as the isovector current associated with excitation of intermediate Δ-isobar resonances. The model for the two-body charge operators is the one of ref. [30] and includes the π-, ρ-, and ω-meson exchange charge operators with both isoscalar and isovector components, the (isoscalar) $\rho\pi\gamma$ and (isovector) $\omega\pi\gamma$ charge transition couplings and the single-nucleon Darwin-Foldy and spin-orbit relativistic corrections.

4 Results

In fig. 4 we show representative results for the transverse response at $q = 600$ MeV/c. The comparison between calculation and experiment shows good agreement in the region of τ, where a significant comparison can be made (as pointed out above, the region $\tau < 0.01$ should be ignored for the case of the transverse response, because it is too sensitive to the tail of the Δ, which was cut away by a finite upper integration limit in ω, see fig. 2).

In particular,

- the large enhancement of the transverse strength due to MEC is correctly predicted,
- the doubling of the transverse excess between $A = 3$ and 4 is reproduced,
- the q-dependence of the transverse excess is correctly accounted for (not shown), and
- the calculation also gives a (small, but non-negligible) *reduction* of the longitudinal strength due to MEC.

Particularly remarkable is the fact that the large excess of the transverse strength is explained by the calculated MEC. Nearly all past calculations of MEC for (e,e') gave quite small contributions in the region of the quasi-elastic peak. When studying the origin of this difference, one finds that MEC give large contributions *only* when the n-p tensor and short-range correlations in both the initial and the final state are included in the calculation (see also [8, 15]). This in most past calculations has not been possible: typically independent-particle wave functions have been used for the initial state, and the final-state interaction (if

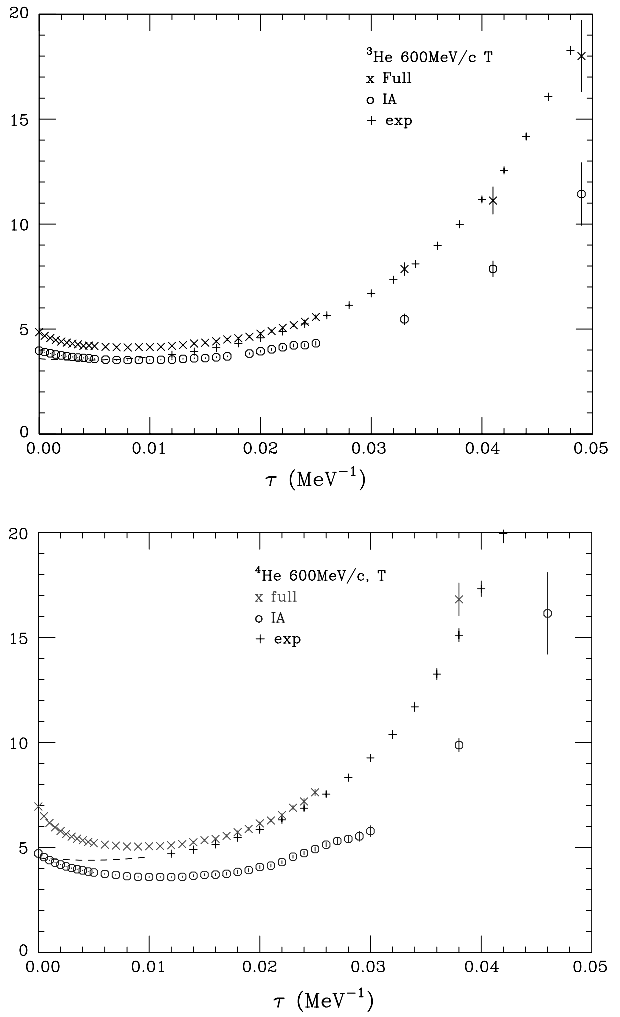

Fig. 4. Transverse Euclidian response for ^{3}He (top) and ^{4}He at 600 MeV/c. Data (+), IA (○) and full calculation (×).

included at all) has been described using an optical potential. The reason for the lack of success of past MEC calculations thus has been elucidated.

5 Sum rules

Sum rules provide a useful tool for the study of integral properties of the response of the nuclear many-body system to the electromagnetic probe. Of particular interest are the ones involving the longitudinal and transverse response functions at constant three-momentum transfer. They can be expressed as ground-state expectation values of the charge and current operators and do not require knowledge of the complicated structure of the nuclear excitation spectrum. These $\tau = 0$ values can more easily (and for more nuclei) be calculated than $E(q,\tau)$.

The sum rules are defined as

$$S_\alpha(q) = C_\alpha \int_{\omega_{\rm th}^+}^{\infty} {\rm d}\omega\, S_\alpha(q,\omega) = C_\alpha \Big[\langle 0|O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q})|0\rangle - |\langle 0|O_\alpha(\mathbf{q})|0\rangle|^2 \Big] \ ,$$

Table 1. The transverse sum rule obtained with one-body only and both one- and two-body current operators.

^{3}He		^{4}He		^{6}Li	
1	1+2	1	1+2	1	1+2
1.01	1.25	1.01	1.49	1.01	1.41

Table 2. The longitudinal sum rule obtained with one-body only and both one- and two-body charge operators.

^{3}He		^{4}He		^{6}Li	
1	1+2	1	1+2	1	1+2
0.982	0.908	0.973	0.910	0.990	0.924

Table 3. The ^{4}He transverse sum rule: contribution of pp and nn pairs.

1	1+2	1+2; pp or nn only
1.01	1.47	1.03

where $S_\alpha(q,\omega)$ is the point-nucleon longitudinal ($\alpha = L$) or transverse ($\alpha = T$) response function, $O_\alpha(\mathbf{q})$ is either the charge $\rho(\mathbf{q})$ or current $\mathbf{j}(\mathbf{q})$ operator divided by the square of the proton form factor $|G_E^p(\tilde{Q}^2)|^2$ ($\tilde{Q}^2$ is evaluated at the energy transfer corresponding to the quasi-elastic peak), $|0\rangle$ denotes the ground state, and the elastic contribution to the sum has been removed. The constants C amount to $C_L = 1/Z$, $C_T = 2m^2/(Z\mu_p^2 + N\mu_n^2)q^2$.

These sums have been calculated [18] for $A = 3$, 4, 6 using the full v_{18} interaction and the MEC discussed above. We here only quote some selected results that let us better understand the agreement between experiment and theory found above.

We first show in table 1, for $q = 600$ MeV/c, the numerical effect of MEC on the transverse strength. The transverse excess is large, increases between $A = 3$ and 4 and only gradually becomes smaller for $A = 6$.

As shown by table 2 MEC and relativistic corrections have also a considerable effect on the longitudinal strength; much of the reduction is due to the Darwin-Foldy term.

When repeating the calculation of the two-body effects with simplified operators, one also finds that the most important two-body current contributions are those associated with the PS (pion-like) and Δ excitation currents.

Moreover, the transverse strength associated with two-body currents is almost entirely due to pn pairs. When only keeping the n-n and p-p correlations, the enhancement in the transverse strength is very small, as shown by table 3, again for $q = 600$ MeV/c. This shows that the enhancement of the transverse response due to MEC is not due to the particular MEC operators used, but is a consequence of the quantitative treatment of the initial- and final-state wave function.

Table 4. Excess strength contributions to the Fermi-gas sum rules from terms involving two-nucleon currents.

ΔS_L	ΔS_T
0.017	0.060

Lastly, we show in table 4 the amount of excess strength due to MEC obtained when removing in the ground state all correlations, *i.e.* when using a Fermi gas. This corresponds to what has been done in most calculations of MEC in the literature [4–17]. Table 4 (again for $q = 600$ MeV/c) largely explains why most previous calculations have found much too small a transverse enhancement.

6 Conclusions

We have discussed the understanding of the separated response functions as measured in inclusive electron-nucleus scattering in the region of the quasi-elastic peak. Much of the discussion of the past had been focused, without good reason as more careful work has shown, on the longitudinal strength (the Coulomb sum rule). The main question not understood, the strong enhancement of the transverse strength in the main quasi-elastic peak region, remained open.

A priori it is clear that the transverse response can get appreciable contributions from meson exchange currents. Actual calculations, however, produced MEC contributions that were far too small. The work discussed in this paper focuses on the transverse strength for ^{4}He, the nucleus where this transverse excess strength is maximal.

It turns out that this excess can be understood once one uses a theoretical approach that does treat the short-range and tensor n-p correlations in both the initial and final (continuum) state. This can be achieved for $A = 3$, 4, 6 as variational Monte Carlo calculations using modern N-N interactions can be performed for the bound states. By studying the Euclidian response (rather than the response as a function of electron energy loss) one can get around the difficulty of a similarly quantitative calculation for the continuum state: the Euclidian response can directly be calculated starting from the ground-state wave function.

The comparison of experimental and calculated response functions shows that both the pronounced enhancement of the transverse strength and the smaller reduction of the longitudinal strength (the Coulomb sum) are due to MEC and can quantitatively be understood.

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